VOLUME 1

MATH TRAVELS

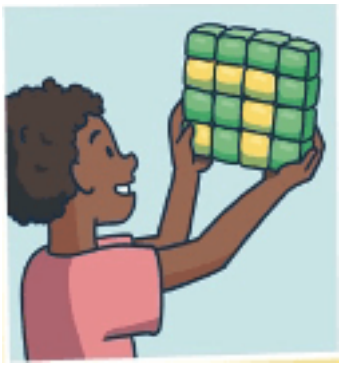
LearnX.ca

George Gadanidis, PhD

MATH PATTERN & BEAUTY

This PDF of *Numbers that Hide in Squares* is a sample from *Math Travels, volume 1*.

Travel to a world of mathematical pattern and beauty!
... with an activity from our research classrooms ...

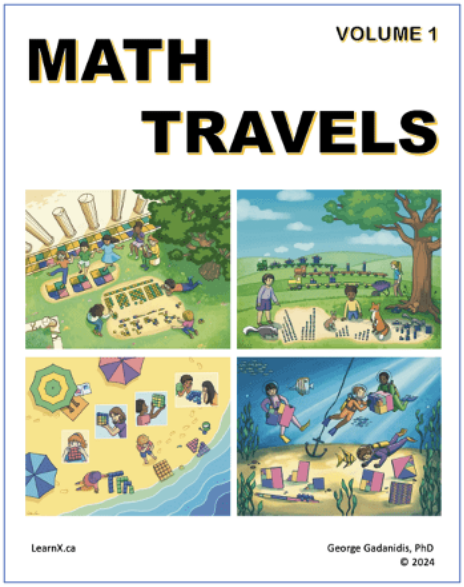


Variations of the *Numbers that Hide in Squares!* activity were implemented in research classrooms & teacher education settings in Ontario, Canada & Rio Claro, Brazil.

This PDF is offered to schools at no cost & with no obligation to purchase resources from LearnX.ca.

$39 – PDF – SCHOOL-WIDE LICENCE

All teachers in your school may use/print this PDF resource!



Passports to 4 math worlds of pattern and beauty!

- 4 Math Travels placemats
  - With activity prompts
- Teaching guide (62 pages)
  - Solutions
  - Coding puzzles
  - Math stories for home connections

Travel to 4 wonderful math worlds, where:

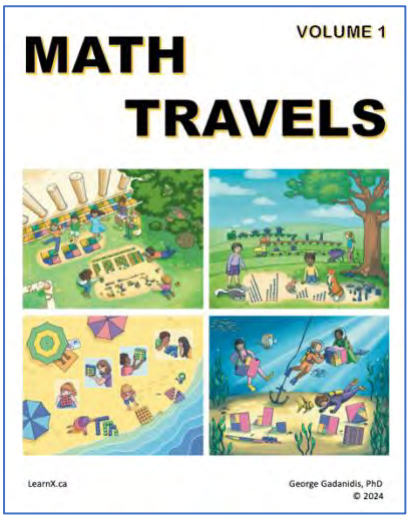
1. Patterns sing & dance
2. Patterns climb staircases
3. Numbers hide in squares
4. Infinity fits in your hand

Experience math concepts and relationships across various representations, in their natural mathematical environments.

MATH TRAVELS – Volume 1

Passports to 4 math worlds of pattern and beauty!

- 4 Math Travels placemats
- Teaching guide
- Solutions
- Coding puzzles
- Math stories for home connections



Travel to 4 wonderful math worlds, where:

1. Patterns sing & dance
2. Patterns climb staircases
3. Numbers hide in squares
4. Infinity fits in your hand

Experience math concepts and relationships across various representations, in their natural mathematical environments.

Inspired by and created for Amelia & Nella.

AUTHOR

George Gadanidis is Professor of Mathematics Education at Western University and Lifetime Fellow of the Fields Institute for Research in Mathematical Sciences.

He has worked as teacher of mathematics, science, and computer science, as mathematics and computer science department lead, as district-wide mathematics coordinator, and as content and digital resource developer for various publishers.

He enjoys co-teaching and co-learning in mathematics research classrooms.

ART

Art by Aileen Lin
Design by George Gadanidis

LEARNX.CA

LearnX.ca is a research dissemination project to inspire students to learn, wonder, and understand (mathematics, coding, AI, and science).

TABLE OF CONTENTS

Page

MATH TRAVELS

Math Travels is a series of *Math Worlds* of pattern and beauty, depicted as travel placemats.

RESEARCH-BASED

Math Travels is based on classroom research of *what might be* when young students engage mathematically with a sense of wonder.

DESIGN PRINCIPLES

Each Math World is original work of art depicting a unique setting, like a beach, an underwater world, or an archaeological dig, to give the aesthetic feel of a new destination and experience.

Math concepts are carefully chosen, represented, arranged, and juxtaposed to reveal *patterns*, elicit *wonder*, offer *low floor* access with minimal prerequisite knowledge, and *high ceiling* depth and complexity.

SEEING-AS

The juxtaposition of math concepts and representations inspires metaphorical thinking – seeing one thing as another – or what Jan Zwicky (2003) calls *seeing-as*.

“All genuine understanding is a form of seeing-as” (p. 3).

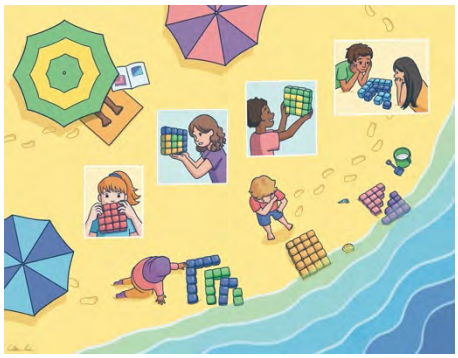
Zwicky, J. (2003). *Wisdom & metaphor.* Kemptville, N.S.: Gaspereau Press.

ATTENTION

What do we mean when we ask students to *pay attention*?

As an analogy, let's consider how movies work, which we not only enjoy attending, but also pay hard cash for the opportunity to do so.

IT'S NEW & WONDERFUL



A good movie offers the "joy of seeing the new and the wonderful" (Boorstin, 1990, p. 12). "People love to be taken to a place that's like nothing they've seen before" (p. 16).

For example, a movie may start with a wide-angle view of the "world" we are about to enter.

The *Math Travels* placemats take us to *Math Worlds* of pattern and beauty.

IT MAKES SENSE

The plot line must be believable, otherwise we think "this can't happen" or "there is no reason for this to happen" and we stop attending. Movie events are organized and make sense in relation to one another.

The 3 images on the right make sense and offer mathematical insights through patterns and conceptual connections in sums of the odd, even, and natural numbers.

Odds: 4x4 or NxN

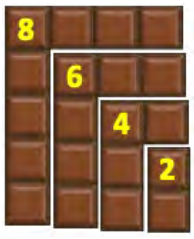
IT SURPRISES

As we watch a movie, we naturally predict what might happen next. If the plot is simple, our predictions are correct. This might feel good the first or second time, but eventually the movie becomes uninteresting.

We don't pay for movies to have our predictions confirmed. We pay for movies to have our predictions challenged, to be wrong, to experience the pleasure of surprise, and to gain new insight (Boorstin, 1990; McKee, 1997).

Odd numbers hide in squares! is a math surprise that spans from patterning in grade 1 to the study of sequences and series in upper high school.

Such math surprises are designed with purpose: to draw attention to math pattern and beauty and to facilitate and anchor understanding.



Evens: 4x5 or N(N+1)

Naturals: (4x5)/2 or N(N+1)/2

Boorstin, J. (1990) *The Hollywood Eye: What Makes Movies Work.* New York, NY: Harper Collins Publishers.

McKee, R. (1997). *Story - substance, structure, style, and the principles of screenwriting*. New York, NY: Harper-Collins/Reagan Books.

HOME CONNECTIONS

Each *Math Travels* placemat, and accompanying math story, offers an opportunity for students to share their learning at home.

IN THE CLASSROOM

Ask students to imagine a family member saying, “What did you do in math today?”

- Discuss in groups and as a whole class:
    - How would you describe what the *Math Travels* placemat is about?
    - What would you share so that they experience a math surprise?
- Script a dialogue.
    - Role play and perform the dialogue as a skit in your group.

Have groups perform their skits to the whole class.

SHARING AT HOME

Ask students to share their learning at home.

They may also take home a print copy of the *Math Travels* placemat and the accompanying math story, as prompts for sharing their learning.

Students should also take home the parent/guardian form (see next page).

Dear Parent/Guardian

We have been working on activities with odd, even, and natural numbers, and their sums.

- Please ask your child to share what they learned and record their responses below.
- Please also record what you learned from this experience.

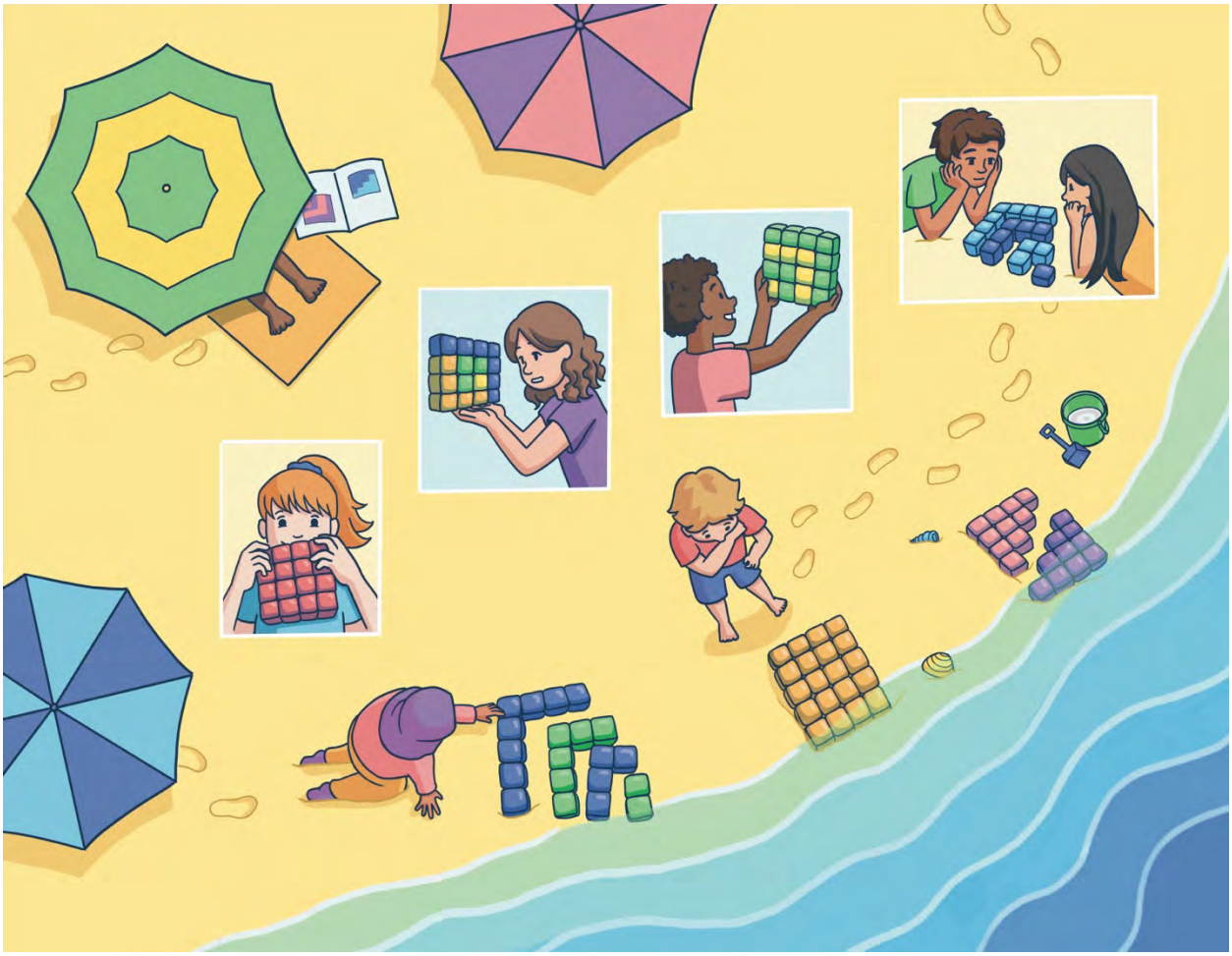
Please ask your child to return this form to me.

Thank you!!

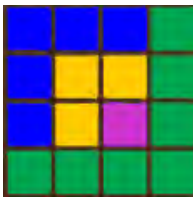
WHAT DID YOUR CHILD SHARE WITH YOU?

WHAT DID YOU LEARN?

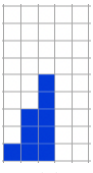
NUMBERS HIDE IN SQUARES (1-8)



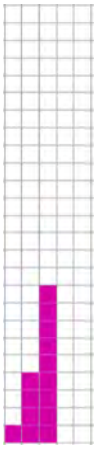
Students in **grades 1-4** investigate growth patterns, describe rules for determining sums of odd and even numbers, and create math art to share their learning at home.



4x4 or NxN



Odds

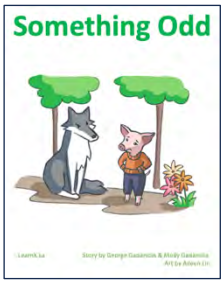


Sums

Students in **grades 6-9** develop algebraic expressions to represent odd, even, and natural numbers, and their sums.

The growth of odd, even, and natural numbers, compared to the growth of their sums, introduces students to linear and non-linear relationships.

Students share the story *Something Odd* and their learning at home.



4x5 or N(N+1)

(4x5)/2 or N(N+1)/2

ACTIVITY 1: What do you see?

1. What do you see?

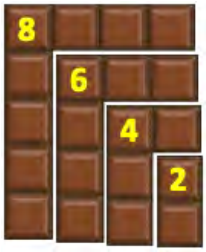
WHAT STUDENTS MAY SEE IN THE PLACEMAT

Students may notice the 3 patterns below, which deal with:

(a) Visual and concrete patterns for consecutive odd, even, and natural numbers.

(b) The resulting numeric and algebraic expressions for their sums – for example, the sum of odd numbers 1 + 3 + 5 + 7 = 4x4 and more generally 1 + 3 + 5 + ... + N = NxN.

Odds: 4x4 or NxN



Evens: 4x5 or N(N+1)

Naturals: (4x5)/2 or N(N+1)/2

ACTIVITY 2: Create *math* art

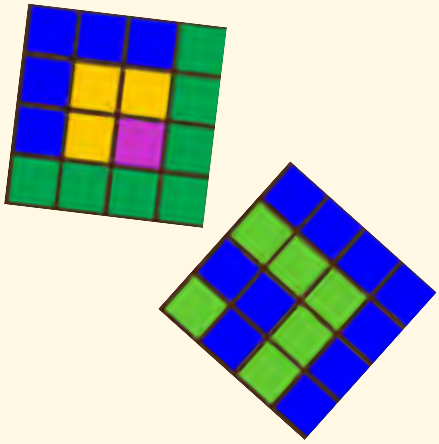
WHAT STUDENTS MAY MAKE

Using linking cubes as well as markers, dabbers, and grid paper, students may create their own colourful and appealing arrangements of odd, even, and natural numbers.

The *math* art may be shared at home, along with practiced dialogues that explain math patterns and concepts:

- Odd numbers hide in squares. For example, the first 4 odd numbers hide in a 4x4 square.
- Even numbers hide in rectangles where one side is one unit larger than the other. For example, the first 4 even numbers hide in a 4x5 rectangle.
- Two sets of natural numbers fit in an even number rectangle.

2. What *math* art might you make?



SCAFFOLDING STUDENT THINKING

Sample scaffolding prompts and questions:

- What math does this art represent?
- How else might you represent it?
- How would you explain it to someone at home, who hasn't seen this before?

ACTIVITY 3: Odd numbers hiding

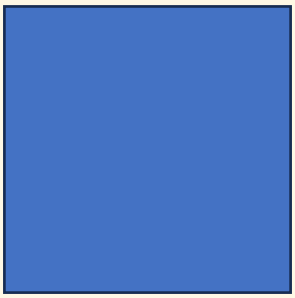
HOW STUDENTS MAY RESPOND

We don't know how many odd numbers are in the blue square.

We may answer in two parts:

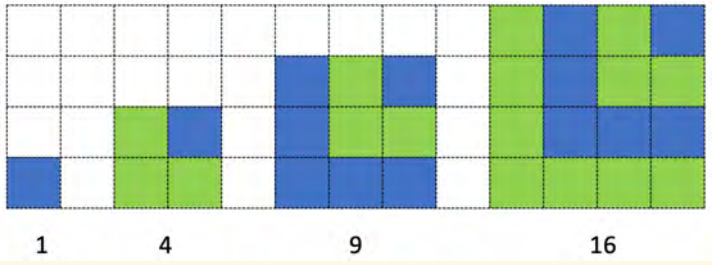
- First, what is the smallest number of odd numbers that may fit in the square?
  - The answer would be 1.
  - The whole square could represent 1 blue cube or the odd number 1.
- Second, what is the largest number of odd numbers that may fit in the square?
  - Let's imagine that the blue square has a 10 x 10 blue grid that is invisible to us.
    - In this grid, we could fit the first 10 odd numbers.
  - If we then imagine drawing horizontal and vertical lines through each grid square, we can create a 20 x 20 grid.
    - In this grid, we could fit the first 20 odd numbers.
  - We can repeat this and keep increasing the number of odd numbers hiding in the square to infinity.

3. How many odd numbers hiding in the square?



ACTIVITY 4: What comes next?

4. How many blocks in the 5^{th} stage?
In the 10^{th} stage?
In the 100^{th} stage?
In the N^{th} stage?



HOW STUDENTS MAY RESPOND

From the pattern above:

- Stage 1 has 1 x 1 = 1 cube
- Stage 2 has 2 x 2 = 4 cubes
- Stage 3 has 3 x 3 = 9 cubes
- Stage 4 has 4 x 4 = 16 cubes
- Stage 5 has 5 x 5 = 25 cubes
- Stage 10 has 10 x 10 = 100 cubes
- Stage 100 has 100 x 100 = 10 000 cubes
- Stage N has N x N = N^2 cubes

In younger grades

We have engaged students as young as grade 1 with such activities.

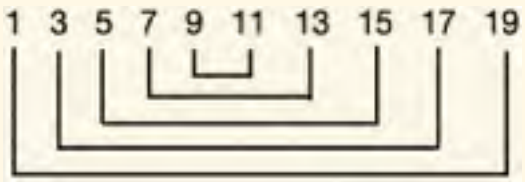
- They successfully complete the patterns up to stage 10.
    - Instead of using multiplication, they typically skip count by 10.
- Some extend these patterns to complete the last 2 questions by describing the result:
    - For stage 100, there are 100 rows of 100 cubes.
    - For stage N, there are N rows of N cubes.

NOTE

N is a variable.
It may represent any stage number.
N x N = N^2 is an expression for the general rule for finding the sum of the first N odd numbers.

ACTIVITY 5: What is this?

5. What is this?



HOW STUDENTS MAY RESPOND

The image above lists the first 10 odd numbers, from 1 to 19, and groups them into 5 pairs, with each pair having a sum of 20.

So, the sum of the first 10 odd numbers 1-19 is 5 x 20 = 100.

We may also use this method to find the sum of the first 10 natural numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

- We may create 5 pairs whose sum is 11: 1+10. 2+9, 3+8, 4+7, 5+6.
- So, the sum of the first 10 natural numbers is 5 x 11 = 55.

Similarly, we may find the sum of the first 10 even numbers: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.

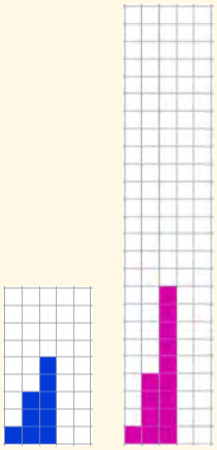
- We may create 5 pairs whose sum is 22: 2+20. 4+18, 6+16, 8+14, 10+12.
- So, the sum of the first 10 even numbers is 5 x 22 = 110.

ACTIVITY 6: How odd sums grow

6. - Plot odd numbers 1-5 as a bar graph.

- Plot sums in stages 1-5 as a bar graph.

- How do they grow?



HOW STUDENTS MAY RESPOND

Odd numbers grow by 2.

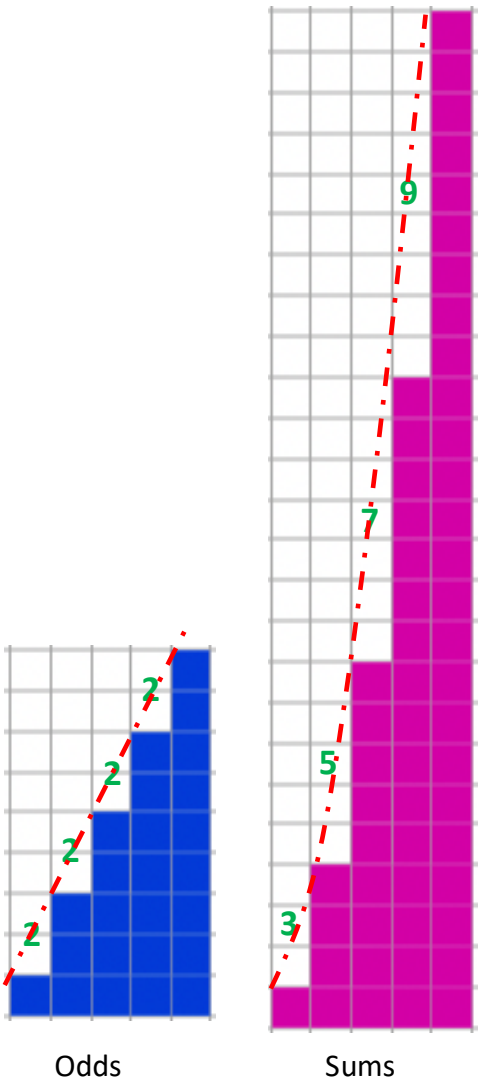
They grow at a constant rate.

If we join the tops of the bars with a red dotted line, it will be straight.

The sum of odd numbers grows by odd numbers: 1, 3, 5, 7, and 9. This makes sense, as the bars represent sums of odd numbers.

The sums grow at an increasing rate.

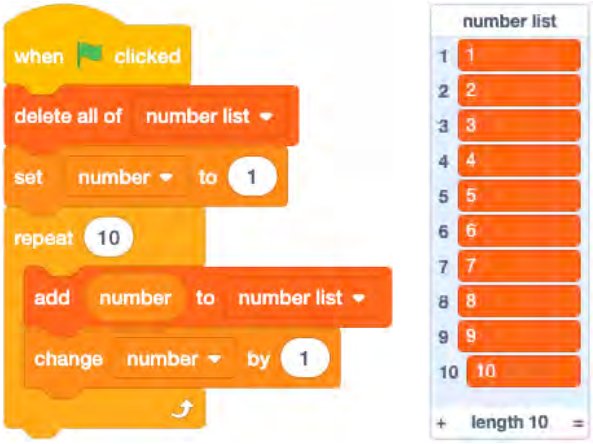
If we join the tops of the bars with a red dotted line, it will be curved.



CODING PUZZLE 1: Number patterns with Scratch

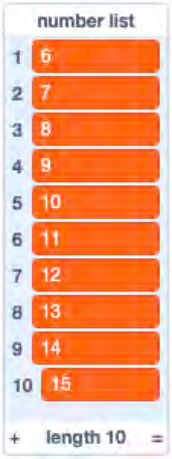
Go to https://scratch.mit.edu/projects/453644929/editor

Run the code. You should get the list of numbers shown below.



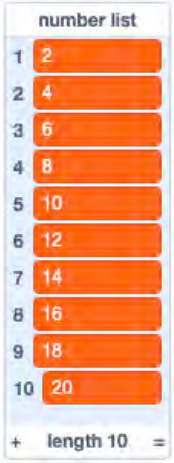
1. Edit the code to get each of the following:

A.

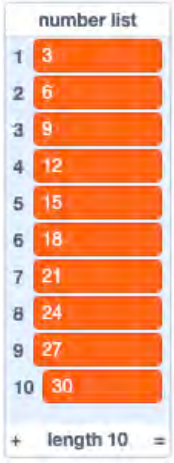


B.

C.



D.



E.

2. Edit the code to list the odd numbers 1-19 in reverse order: 19, 17, 15, 13, 11, 9, 7, 5, 3, 1.

SOLUTIONS

1.A. set number to 6
1.B. set number to 1; change number by 2
1.C. set number to 2; change number by 2
1.D. set number to 3; change number by 3
1.E. set number to 500; change number by 500

2. set number to 19; change number by -2

CODING PUZZLE 2: Number sums with Scratch

1. Go to https://scratch.mit.edu/projects/457027214/editor
2. Run the code. You should get the lists of numbers and sums shown below.

	number list	sum list
1	1	1
2	2	3
3	3	6
4	4	10

3. Edit the code to list and find the sum of numbers 10, 20, 30, and 40. Can you predict their sum?
4. Edit the code to list and find the sum of numbers 100, 200, 300, and 400. Can you predict their sum?
5. Edit the code to list and find the sum of numbers 11, 22, 33, and 44. Can you predict their sum?
6. Edit the code to list and find the sum of numbers 111, 222, 333, and 444. Can you predict their sum?

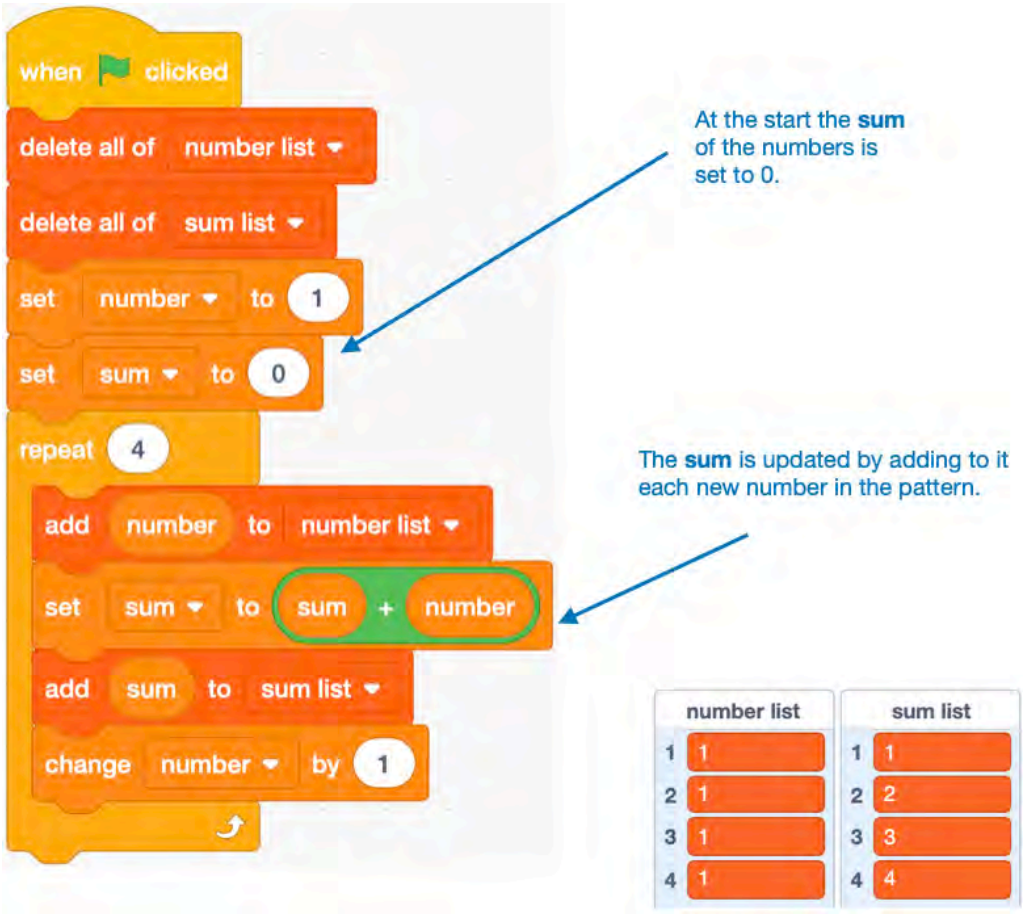
SOLUTIONS

3. To print multiples of 10, make these edits:
   a. Set number to **10**
   b. Change number by **10**
4. To print multiples of 100, make these edits:
   a. Set number to **100**
   b. Change number by **100**
5. To print multiples of 11, make these edits:
   a. Set number to **11**
   b. Change number by **11**
6. To print multiples of 111, make these edits:
   a. Set number to **111**
   b. Change number by **111**

HOW PUZZLE 2 CODE WORKS

The code calculates and lists the natural numbers and their sums.

- Using **set number to 1** and **change number by 1**, the code calculates and lists the numbers 1-4.
- Using **set sum to 0** and **set sum to sum + number**, the code calculates and lists the sums of the numbers 1-4.



VARIATIONS

1. To list and add odd numbers, edit **change number by 1** to **change number by 2**.
2. To list and add even numbers, edit **set number to 1** to **set number = 2** and **change number by 1** to **change number by 2.**

CODING PUZZLE 3: Number patterns with Python

EVEN NUMBERS

1. Go to cscircles.cemc.uwaterloo.ca/console and enter and run the code below. It will list the first 5 even numbers.

```
for N in range (1,6):
    print (2*N)
```

```
2
4
6
8
10
```

   A. Edit the code to list the first 10 even numbers.

   B. Edit the code to list the even numbers 6 to 20.

ODD NUMBERS

2. Edit the code to list:

   A. The first 5 odd numbers

   B. The odd numbers 11 to 25

NATURAL NUMBERS

3. Edit the code to list:

   A. The first 10 natural numbers

   B. The natural numbers 10 to 20

SOLUTIONS

1A

```
for N in range (1,11):
    print (2*N)
```

1B

```
for N in range (3,11):
    print (2*N)
```

2A

```
for N in range (1,6):
    print (2*N-1)
```

2B

```
for N in range (6,14):
    print (2*N-1)
```

3A

```
for N in range (1,11):
    print (N)
```

3B

```
for N in range (10,21):
    print (N)
```

CODING PUZZLE 4: Number sums with Python

NATURAL NUMBERS & THEIR SUMS

1. Go to cscircles.cemc.uwaterloo.ca/console
2. Enter and run the code below. It will list the first 4 natural numbers and their sums.

```
number = 1
SUM = 0
for counter in range(1,5):
    SUM = SUM + number
    print (number, SUM)
    number = number + 1
```

```
1 1
2 3
3 6
4 10
```

3. Edit the code to find the sum of numbers 10, 20, 30, and 40. Can you predict their sum?
4. Edit the code to find the sum of numbers 100, 200, 300, and 400. Can you predict their sum?
5. Edit the code to find the sum of numbers 11, 22, 33, and 44. Can you predict their sum?
6. Edit the code to find the sum of numbers 111, 222, 333, and 444. Can you predict their sum?

SOLUTIONS

3

```
number = 10
SUM = 0
for counter in range (1,5):
    SUM = SUM + number
    print (number, SUM)
    number = number + 10
```

```
10 10
20 30
30 60
40 100
```

4

```
number = 100
SUM = 0
for counter in range (1,5):
    SUM = SUM + number
    print (number, SUM)
    number = number + 100
```

```
100 100
200 300
300 600
400 1000
```

5

```
number = 11
SUM = 0
for counter in range (1,5):
    SUM = SUM + number
    print (number, SUM)
    number = number + 11
```

```
11 11
22 33
33 66
44 110
```

6

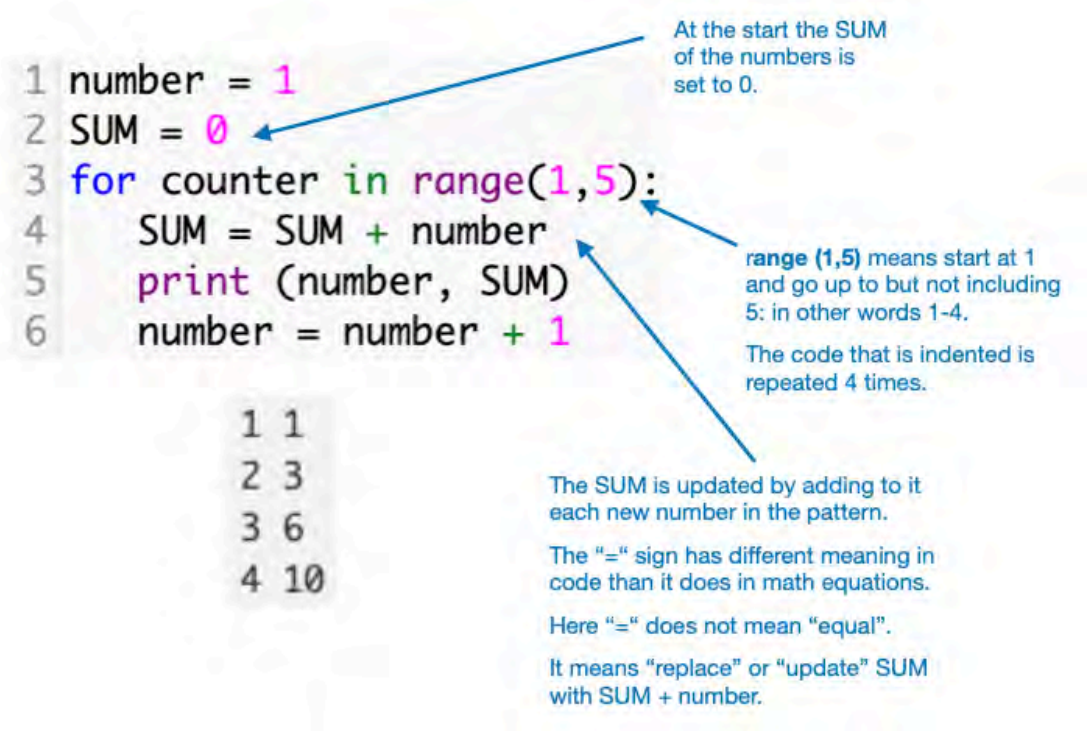
```
number = 111
SUM = 0
for counter in range (1,5):
    SUM = SUM + number
    print (number, SUM)
    number = number + 111
```

```
111 111
222 333
333 666
444 1110
```

HOW PUZZLE 4 CODE WORKS

The code calculates and prints natural numbers and their sums.

- Using **number = 1** and **number = number + 1**, the code calculates and lists the numbers 1-4.
- Using **SUM = 0** and **SUM = SUM + number**, the code calculates and lists the sums of the numbers.



VARIATIONS

1. To list and add odd numbers, change **number = number + 1** to **number = number + 2.**
2. To list and add even numbers, change **number = 1** to **number = 2**, and **number = number + 1** to **number = number + 2.**

MATH STORY: *Something Odd*

The following illustrated story is a playful introduction to odd numbers and their sums.

Something Odd

Story by George Gadanidis & Molly Gadanidis
Art by Aileen Lin

Piggy's mom warned: "Watch out for big bad Wolf!"

But Piggy found wolves curious and imagined them loping through the woods, or standing hunched, staring at the far distance through their eyebrows.

So, one day, as Piggy cooked some sloppy slop in the kitchen of his somewhat sturdy house of straw, he left the door open, just a tiny, tiny bit.

"It's too hot in my kitchen," he reasoned with himself. "A fresh breeze will come through the door."

But the breeze brought in a stinky surprise, and Piggy turned to see Wolf huffing and puffing at the door!

"I have a surprise for you," said Wolf. "Come and see."

"My mom said to watch out for you," replied Piggy, a little scared.

"No need to be afraid," said Wolf, his paw extended, a chocolate bar in his grasp.

"That's odd," thought Piggy, for the chocolate bar was square.

"The store-bought bars are always rectangular," he thought, as he eyed the chocolate greedily.

Unable to resist, Piggy studied the 5-by-5 arrangement. “25 pieces,” he blurted to Wolf.

“There’s a secret hiding inside,” tempted Wolf, as he extended his paw further.
“Take a piece,” he offered.

Despite mom’s warnings, Piggy was curious where this might lead. He reached out, as if leaning over the edge of a cliff, and snapped off 1 piece.

“Take another,” enticed Wolf, his stare steady through scraggly eyebrows.

Piggy reached again. The piece that broke off was bigger this time: 3 pieces in the shape of an L.

“Take one more, and tell me the secret of the square,” smiled Wolf, as much as wolves can smile.

An even bigger piece broke off next: 5 pieces in a bigger L.

“I know what the next piece will be!” said Piggy excitedly. “7 pieces,” he added, as he broke off an even bigger L.

Piggy looked at the last L left on Wolf’s paw. “There are odd numbers hiding in squares?” he said, half surprised, half curious, half excited, and a little bit scared.

“I wonder what shapes even numbers hide in?” asked Piggy.

“That’s a delicious question,” said Wolf, rubbing his rumbling tummy. He then loped away, disappearing into the woods.

Piggy couldn’t wait to tell his older sister about his math adventure.

But, as he approached his sister’s house, Piggy once again sensed something *odd*.

THE END